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BAND OF SELECTIVE TRANSMISSION OF ELECTROMAGNETIC RADIATION BY AN ABSORBING DIELECTRIC LAYER

É. R. Kasimov

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Equations for determining the frequency band near the frequency of selective reflectionless transmission of electromagnetic radiation by a layer of an absorbing dielectric which separates two nonabsorbing media with dissimilar optical densities have been obtained. The absorption band of the wave in the dielectric layer as a function of the dielectric properties and thickness of the layer and of the dielectric properties of the separated media has been evaluated.

The conditions of occurrence and the region of existence of the reflectionless transmission of electromagnetic radiation by a plane layer of an absorbing dielectric which separates two semiinfinite nonabsorbing media have been found in [1]. One important characteristic of such selective transmission of a wave by the absorbing layer is a frequency band within which the reflection of the wave is minimum and does not exceed the prescribed allowable value. Evaluation of the frequency band and of its dependence on the thickness of the layer and its dielectric properties and on the properties of the media adjacent to the layer are of practical significance in creating narrow-band absorbing optical and microwave devices.

To determine the band of selective transmission of electromagnetic radiation by an absorbing layer we consider the problem of reflection of a plane-parallel electromagnetic wave incident perpendicularly on a plane absorbing-dielectric layer adjustable for thickness; the absorbing dielectric separates two semiinfinite nonabsorbing media with dissimilar optical densities. The substance of the absorbing layer has the complex value of the permittivity $\varepsilon = \varepsilon' - i\varepsilon''$, while the media adjacent to it have permittivities ε_1 and ε_2 respectively. In the case of incidence of the wave from the medium with a value ε_1, the complex value of the coefficient of reflection of the wave from the two-layer system consisting of the absorbing layer and the medium with a value ε_2 is equal to

$$\dot{\rho} = \frac{Z_{\text{in}} - Z_1}{Z_{\text{in}} + Z_1}, \tag{1}$$

where $Z_{\text{in}} = Z \dfrac{Z_2 + Z \tanh \gamma l}{Z + Z_2 \tanh \gamma l}$ is the input resistance of the two-layer system; $\gamma = i \dfrac{2\pi}{\lambda} \sqrt{\varepsilon}$ [2].

We introduce the notation $x = l/\lambda_{\text{d}}$ and allow for the known expressions for the relationship between the dielectric and optical properties of dielectrics

$$\varepsilon' = n^2 (1 - y^2), \quad \varepsilon'' = 2n^2 y, \quad \varepsilon_1 = n_1^2, \quad \varepsilon_2 = n_2^2, \tag{2}$$

where $y = \tan \dfrac{\delta}{2}$, $\delta = \arctan \dfrac{\varepsilon''}{\varepsilon'}$.

We assume that

$$\tanh 0.5 (\alpha_1 + i\beta_1) = \frac{Z_1}{Z}, \quad \tanh 0.5 (\alpha_2 + i\beta_2) = \frac{Z_2}{Z}. \tag{3}$$

Since $Zn(i - iy) = Z_1 n_1 = Z_2 n_2 = Z_0$, we have

Institute of Photoelectronics, National Academy of Sciences of Azerbaijan, Baku, Azerbaijan; email: eminka@azeuro.net. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 1, pp. 110–113, January–February, 2003. Original article submitted February 26, 2002; revision submitted May 8, 2002.

$$\alpha_1 = -\ln r_1\,, \quad \beta_1 = \arctan \frac{2nn_1y}{n^2(1+y^2)-n_1^2}\,, \quad r_1 = \sqrt{\frac{(n_1-n)^2+(ny)^2}{(n_1+n)^2+(ny)^2}}\;;$$

$$\alpha_2 = -\ln r_2\,, \quad \beta_2 = \arctan \frac{2nn_2y}{n^2(1+y^2)-n_2^2}\,, \quad r_2 = \sqrt{\frac{(n_2-n)^2+(ny)^2}{(n_2+n)^2+(ny)^2}}\;, \tag{4}$$

where r_1, β_1, r_2, and β_2 are, respectively, the moduli and phases of the coefficients of reflection of the wave from the boundary of the absorbing layer with the first and the second media adjacent to it.

We represent the expression for the input resistance of the two-layer system Z_{in} in the following reduced form:

$$Z_{in} = Z_1(E+iF) = \tanh(\varphi_1 + i\varphi_2)\,, \tag{5}$$

where $\varphi_1 = 2\pi xy + \alpha_2$ and $\varphi_2 = 2\pi x + \beta_2$. The real and imaginary parts of the reduced input resistance of the system will be equal, respectively, to

$$E = \frac{n_1}{n(1+y^2)} \frac{\sinh 2\varphi_1 - y\sin 2\varphi_2}{\cosh 2\varphi_1 + \cos 2\varphi_2}\,, \quad F = \frac{n_1}{n(1+y^2)} \frac{y\sinh 2\varphi_1 + \sin 2\varphi_2}{\cosh 2\varphi_1 + \cos 2\varphi_2}\,. \tag{6}$$

The reflectionless transmission of electromagnetic radiation by the absorbing layer corresponds to the conditions $\dot{\rho} = 0$, $Z_{in} = Z_1$, $E = 1$, and $F = 0$ with allowance for which from Eqs. (6) it follows that

$$\frac{n(1+y^2)}{n_1} = \tanh \varphi_1 - y\tan \varphi_2\,, \tag{7}$$

$$y\sinh 2\varphi_1 + \sin 2\varphi_2 = 0\,. \tag{8}$$

The dependence of the modulus of the reflection coefficient of the wave ρ on the thickness of the absorbing layer l of the system in question represents an oscillating and decaying curve. Therefore, it may be assumed that the reflectionless transmission of the wave will occur when the layer thickness corresponds to one minimum of the function $\rho(l)$ but on condition that the quantity ρ at this minimum reaches its zero value. It is assumed that any of the minima of ρ, including the zero minimum mentioned, is realized when the layer thicknesses are equal to

$$x = \frac{l}{\lambda_d} = \frac{ln}{\lambda} = \frac{(2N-1)}{4} + \Delta\,, \tag{9}$$

where N is the number of the zero minimum of the function $\rho(l)$ and Δ is a small but nonzero quantity dependent on the number of the minimum and on the dielectric properties of the coating and the substrate.

From simultaneous solution of Eqs. (7) and (8) with account for relation (9) and expressions (4) we have

$$\pi(2N-1) + 4\pi\Delta = \frac{1}{y}\ln\frac{r_2}{r_1}\,, \tag{10}$$

where

$$\Delta = \frac{1}{4\pi}[\beta_1 - \beta_2]\,. \tag{11}$$

Equation (10) determines functional relationships between such selective values of n, n_1, n_2, and y and consequently, in accordance with Eqs. (2), also between ε', ε'', ε_1, and ε_2 for which the conditions of existence of the reflectionless transmission of the wave by the absorbing layer are fulfilled. The necessary thickness of the layer is determined from Eqs. (9) and (11).

To determine the wavelength band $\Delta\lambda$ within which the value of ρ is nonzero and does not exceed a certain quantity ρ_b we will assume that the boundary value ρ_b is low and the dielectric properties of the absorbing layer and of the media adjacent to it do not change with frequency. Then, as has been shown in [3], the wavelength band $\Delta\lambda$ near the obtained selective wavelength λ will be determined by the equation

$$\Delta\lambda = \frac{\rho_b}{\sqrt{(E_0')^2 + (F_0')^2}}, \tag{12}$$

where E_0' and F_0' are, respectively, the derivatives of the real and imaginary parts of the reduced input resistance of the system for the selective values of its parameters.

Using the relations (6) obtained for E and F, we will have

$$\frac{\Delta\lambda}{\rho_b\,\lambda_0} = \frac{\sinh\,(4\pi x_0 y_0 - \ln r_2)}{\pi x_0}, \tag{13}$$

where $x_0 = l_0 n_0/\lambda_0$ and λ_0, l_0, n_0, and y_0 are the selective values of the wavelength, the layer thickness, the refractive index, and the dielectric loss factor of the substance layer.

When the values of N are high, we can disregard the quantity Δ in Eqs. (9) for x_0 because of its smallness as compared to N and can use the approximate equation

$$\frac{\Delta\lambda}{\rho_b\,\lambda_0} = \frac{4\sinh\,[\pi\,(2N-1)\,y - \ln r_2]}{\pi\,(2N-1)}. \tag{14}$$

In the particular case where the material of the separating dielectric layer does not absorb electromagnetic radiation we have $y_0 = 0$, $\Delta = 0$, and the fulfillment of the following conditions of optics with an anti-reflection coating: $l_0 = (2N-1)\,\lambda_d/4$ and $n = \sqrt{n_1 n_2}$ [4]. Then Eq. (3) is reduced to the form

$$\frac{\Delta\lambda}{\rho_b\,\lambda_0} = \frac{8n_0}{\pi\,(2N-1)\,(n_0^2-1)}, \tag{15}$$

which coincides with the equation obtained in [5].

If we use metal as the substance of the second medium, we obtain $n_2 \to \infty$ and $r_2 \to 1$. As a result, Eq. (13) yields the relation

$$\frac{\Delta\lambda}{\rho_b\,\lambda_0} = \frac{\sinh\,[4\pi x_0 y_0]}{\pi x_0}, \tag{16}$$

which coincides with the equation obtained in [3] for the absorption band of a wave incident on the two-layer system dielectric–metal.

Equations (7)–(16) determine the existence conditions and the band of reflectionless transmission of electromagnetic radiation by the absorbing dielectric layer irrespective of the dielectric properties of the media separated by it. For convenience of presentation of the results obtained we have considered two variants of solution of Eqs. (7)–(16) which correspond to the transmissions of the wave by the layer from an optically less dense medium to a denser medium: the direct variant of the problem for $n_1 < n_2$ and the inverse variant for $n_1 > n_2$.

Figure 1 shows the relationships between the reduced values of the relative transmission band of the wave $\Delta\lambda/\rho_b\lambda_0$ and the selective values of n, n_1, and n_2 of the substances of the absorbing layer and the media adjacent to it, which are calculated by Eqs. (13)–(16). The dependences are given for the first two zero minima of the function $\rho(l)$ respectively for the direct (a) and inverse (b) variants of consideration of the problem of transmission of the wave by the layer. In constructing them, we have used the relative values n/n_1 and n_2/n_1 (see Fig. 1a) and n/n_2 and n_1/n_2 (see Fig. 1b). The obtained dependences exist in the intervals of variation of the quantities n, y, n_1, and n_2 for which the conditions of reflectionless transmission of the wave by the absorbing layer determined by Eqs. (7) and (8) can be fulfilled. These intervals are respectively $(0,\ n_b/n_1)$ for the direct variant and $(n_b/n_2,\ \infty)$ for the inverse variant

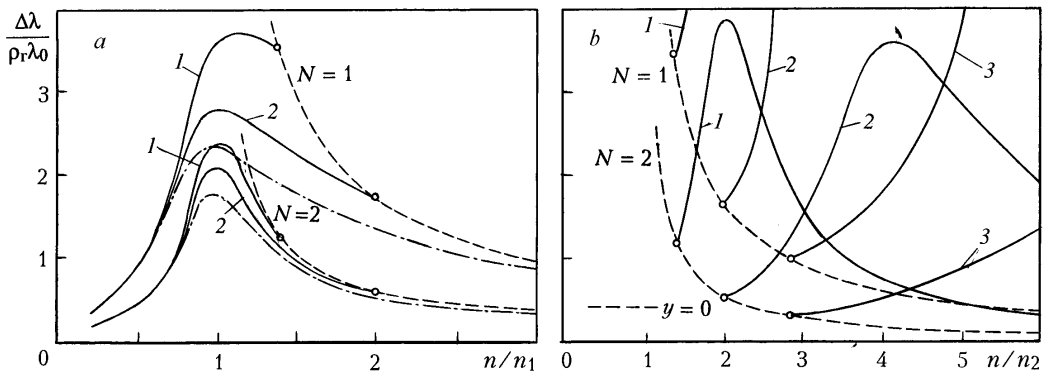

Fig. 1. Relationships between the values of the relative band of quenching $\Delta\lambda/\lambda_0$ of electromagnetic radiation and the selective values of the refractive indices n, n_1, and n_2 of the absorbing layer and the media adjacent to it respectively in the case of reflectionless transmission, by the absorbing layer, of the wave from an optically less dense medium to an optically dense medium (a) and from an optically dense medium to a less dense medium (b) (dashed curves, $y = 0$; dash-dot curves, metal substrate): a) 1) $n_2/n_1 = 2$ and 2) 4; b) 1) $n_1/n_2 = 2$, 2) 4, and 3) 8.

of solution of the problem of transmission of the wave by the absorbing dielectric layer. The boundary value is $n_b = \sqrt{n_1 n_2}$ and it is determined from the condition of making the nonabsorbing separating layer anti-reflecting [4].

When electromagnetic radiation from an optically less dense medium is transmitted to a denser medium (direct variant of solution of the problem), the value of the relative absorption band $\Delta\lambda/\lambda_0$ decreases with increase in N and in the ratio n_2/n_1 for the prescribed selective values of λ_0 and l_0. The value of the band increases with n and passes through the maximum near $n = n_1$. The position of this maximum depends weakly on N and the relation between n_1 and n_2 (see Fig. 1a). For a prescribed N the dependences obtained are bounded in the coordinate plane by the limiting curves shown dashed in Fig. 1a. Of them, the lower curve, described by Eq. (15), corresponds to the case where a metal substrate has been used as the second medium, while the upper curve, described by Eq. (16), corresponds to the case of the absence of absorption in the substance of the separating layer ($y = 0$). Thus, the value of the absorption band in the system in question is always higher than that in the case of deposition of the separating layer on the metal substrate but it is lower in the absence of absorption in this layer. These differences become smaller as the selective values of n increase and the compared curves themselves in the limit asymptotically approach each other for high values of n.

The dependences of the relative absorption band on n and N are also similar in character in the case where electromagnetic radiation is transmitted by the layer from the optically dense medium to a less dense medium (inverse variant of solution of the problem). The value of the band decreases with increase in N and in the ratio n_1/n_2. The value of the band increases for high values of n and passes through the maximum near $n = n_1$ (see Fig. 1b). The positions of this maximum depend weakly on N and the relation between n_1 and n_2. However, unlike the results of solution of the direct problem, for a prescribed N the dependences of $\Delta\lambda/\rho_b\lambda_0$ on n/n_2 obtained for the inverse problem lie higher than the limiting curves corresponding to the particular case of the absence of absorption in the separating layer. These differences are significant when $n = n_1$; they become smaller with further increase in n, and the compared curves themselves asymptotically approach each other for high values of n.

The regularities established in determining the conditions and the frequency band of reflectionless transmission of electromagnetic radiation by an absorbing layer can be applied to the creation of narrow-band absorbing devices of optical and high-frequency wave ranges.

NOTATION

l, thickness of the separating absorbing layer; N, number of the zero minimum of the reflected wave; Z_0, Z, Z_1, and Z_2, wave resistances of vacuum and of the substances of the absorbing layer and the adjacent media; $\dot{\rho}$ and ρ, complex value and modulus of the reflection coefficient of the wave respectively; ρ_b, the same at the boundary of the band of selective transmission of the wave; $\Delta\lambda$, band of selective transmission of the wave; γ, propagation constant of the wave in the substance of the absorbing layer; n, y, ε', ε'', and δ, refractive index, dielectric loss factor, permittivity, dielectric loss, and dielectric loss angle of the substance of the absorbing layer; n_1, n_2, ε_1, and ε_2, refractive index and permittivity of the materials of the media adjacent to the layer; λ and λ_d, wavelength in vacuum and in the absorbing-layer substance. Subscripts: 0, for the cases of reflectionless transmission of electromagnetic radiation by the absorbing layer; in, input; d, dielectric; b, boundary value.

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